CE 228N: Introduction to the Theory of Plasticity: Homework I

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Make sure to indicate the Burgers vector, the dislocation line, and the direction of the external shear stress as required. Use Michell's table of Airy stress functions where needed. For the coding problem, the source code must be included in your pdf. All plots should be labeled clearly.

1. Use a sequence of 3D sketches to show how a screw dislocation can produce a slip step.

2. Show the atomic positions in and around a screw dislocation by considering multiple, stacked atomic planes. Make a Burgers circuit around the dislocation and identify the Burgers vector $\mathbf{b}$. Verify that $\mathbf{t} \,||\, \mathbf{b}$.

3. For the Volterra 'glide' edge dislocation problem that was worked out in class, verify that the stress field corresponding to the Airy stress function $2r \log r \sin\theta$ is self-equilibrated.

4. Show that the Kolosov stress-strain formulae for the normal strain components in planar elasticity

$$\epsilon_{rr} = \left(\frac{\kappa+1}{8\mu}\right)\sigma_{rr} - \left(\frac{3-\kappa}{8\mu}\right)\sigma_{\theta\theta}$$
$$\epsilon_{\theta\theta} = \left(\frac{\kappa+1}{8\mu}\right)\sigma_{\theta\theta} - \left(\frac{3-\kappa}{8\mu}\right)\sigma_{rr}$$

   reduce to the plane-strain constitutive relations when the Kolosov constant $\kappa = 3 - 4\nu$. Show that the value of κ to reproduce plane stress constitutivity is $\kappa = (3-\nu)/(1+\nu)$.

5. Find the stress field for an elastic climb dislocation in an isotropic linear elastic solid with displacement discontinuity b along the positive x-axis. The material has elastic moduli G and ν. The line of the dislocation is the z-axis.

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6. Using the anti-plane shear idea, show that the stress field of a right-handed screw dislocation whose dislocation line is the z-axis is $\sigma_{\theta z} = \dfrac{Gb}{2\pi r}$. Hence, find an expression for the elastic strain energy density per unit length of a screw dislocation.

7. For the three-element Standard model (Kelvin network in series with a spring E_1), show that

$$\dot{\sigma} + \left(\frac{E_1 + E_2}{\eta}\right)\sigma = E_1\dot{\varepsilon} + \frac{E_1 E_2}{\eta}\varepsilon$$

8. Derive a similar equation for an alternative three-element network (Maxwell network in parallel with a spring E_1).

9. Consider a three-element overstress viscoplastic network with constants E, η, and σ_Y. Assume that the network is subjected to a tensile strain $\varepsilon_0 > \sigma_Y/E$ that is applied suddenly, at time $t = 0$, and then held constant for $t > 0$. Find and plot the stress response $\sigma(t)$. How does the corresponding Maxwell network (with identical E, η) respond to the same applied strain? Comment on the ability of these networks to exhibit stress relaxation.

10. Simulation problem *(MATLAB is recommended for this coding exercise.)*
    Consider an overstress viscoplastic network with constants $E = 100$ MPa, $\sigma_Y = 0.2$ MPa and relaxation time $\tau = \eta/E = 0.5$ s. Assume that the network is subjected to the following 3-step loading sequence:

    (a) A linear ramp-up in stress from 0 to $\sigma_0 = 1.8\,\sigma_Y$ in a time $T_{ramp} = 2$ sec.

    (b) The applied stress is then held at σ_0 for a time $T_{hold} = 5$ sec.

    (c) A linear ramp-down in stress from σ_0 to 0 in a time $T_{fade} = 5$ sec.

    Write code to analyze the response of this network and do the following:

    (a) Find the total strain $\varepsilon(t)$, the viscoplastic strain $\varepsilon_{vp}(t)$, and the elastic strain $\varepsilon_e(t)$. Plot all three strain histories in one figure.

    (b) Plot the stress-strain $\{\sigma(t)\text{-}\varepsilon(t)\}$ response for this problem.

    (c) What is the change in slope at $\sigma = \sigma_Y$?

    (d) What is the additional (visco) plastic strain developed during unloading? What is the residual viscoplastic strain?

    (e) How do the results in parts (a), (b) and (d) change when (i) $\tau = 0.1$ sec (ii) $\tau = 10$ sec?

    You might have to solve the problem incrementally, i.e. step through the stress history using a small time increment Δt. Report the number of increments and justify your choice of Δt.